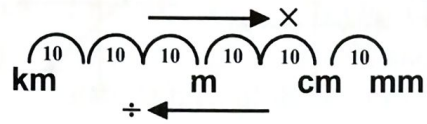
CHAPTER 16: MEASUREMENT

TOPIC: SURFACE AREA AND VOLUME OF 3D SHAPES

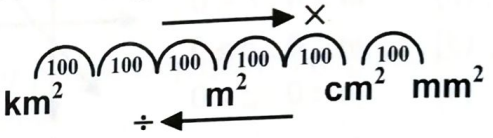
In Grade 8, you studied the surface area and volume of cubes, rectangular prisms (cuboids) and triangular prisms. We will revise this work and then focus on the surface area and volume of cylinders.

However, before proceeding with any further, let's revise converting between units (lengths, areas, volumes and capacities. The Grade 8 textbook explains how to use these diagrams.

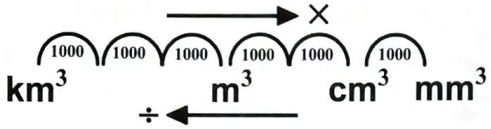
Converting between lengths



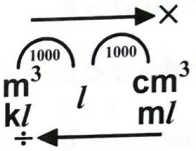
Converting between areas



Converting between volumes



Converting between capacities



1 m*l* = 1 cm³

1 k*l* = 1 m³

EXERCISE 1

(a) Convert:

(1) 6 km to m. (2) 0,06 m to cm (3) 60 cm to mm
(4) 80 mm to cm (5) 0,08 cm to m (6) 80 m to km
(7) 70 000 cm to km (8) 7 mm to m (9) 0,07 mm to cm

(b) Convert:

(1) 9 km^2 to m^2 (2) 0,009 m^2 to cm^2 (3) 30,4 cm^2 to mm^2
(4) 2 500 mm^2 to cm^2 (5) 45 cm^2 to m^2 (6) 80 000 m^2 to km^2
(7) 500 000 cm^2 to km^2 (8) 65 000 mm^2 to m^2 (9) 1,25 km^2 to m^2

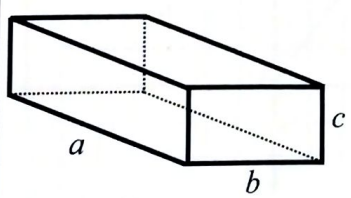
(c) Convert:

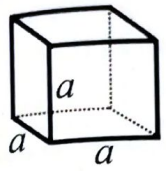
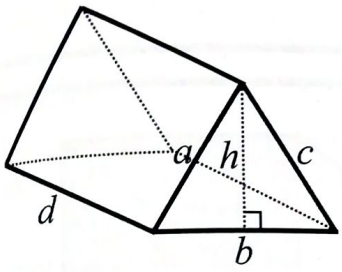
(1) 4 cm^3 to mm^3 (2) 0,002 m^3 to cm^3 (3) 0,002 km^3 to m^3
(4) 0,000 009 m^3 to mm^3 (5) 7 600 000 m^3 to km^3 (6) 750 mm^3 to cm^3
(7) 9 000 000 cm^3 to m^3 (8) 0,000 000 000 000 005 km^3 to mm^3

(d) Convert:

(1) 30 k*l* to *l* (2) 3 k*l* to m*l* (3) 0,003 *l* to m*l*
(4) 70 000 *l* to k*l* (5) 70 m*l* to *l* (6) 8 000 m*l* to k*l*
(7) 40 m*l* to cm^3 (8) 40 k*l* to m^3 (9) 6 *l* to cm^3
(10) 0,018 k*l* to cm^3 (11) 13 000 cm^3 to *l* (12) 13 000 cm^3 to k*l*
(13) 0,123 m^3 to *l* (14) 0,123 m^3 to m*l* (15) 0,123 k*l* to m^3

Revision of the Grade 8 formulae for calculating surface area and volume

Prism	Surface area	Volume
Cuboid (Rectangular prism)  	Sum of the areas of the six rectangles: Surface area $= 2ab + 2ac + 2bc$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $= (ab) \times c$ $= abc$

Cube  	Sum of the areas of the six squares: Surface area $=2(a)(a)+2(a)(a)+2(a)(a)$ $=6a^2$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $=(a\times a)\times a$ $=a^3$
Triangular prism  	Sum of the areas of two triangles and three rectangles. Surface area $=2\left[\frac{1}{2}(b\times h)\right]+ad+bd+cd$ $=bh+ad+bd+cd$ $=bh+d(a+b+c)$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $=\frac{1}{2}(b\times h)\times d$ $=\frac{1}{2}bhd$

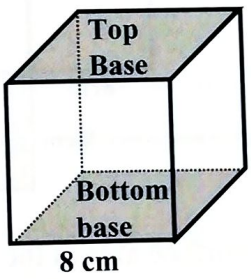
Note: The surface area and volume of a prism can be calculated without these formulae. It is not necessary to learn them off by heart. It is highly recommended that you understand how to calculate surface area and volume using common sense rather that learning formulae off by heart without understanding how they work.

The surface area of a cube, cuboid and triangular prism (Revision)

A prism is a three-dimensional shape with two congruent parallel polygonal faces at opposite ends. These faces are referred to as the **bases** (or ends) of the prism. In a cube, the faces are squares and in a rectangular prism (cuboid), the faces are rectangles. A triangular prism is made up of two congruent triangles and three rectangles. One of the triangles is the base.
The **surface area** of a three-dimensional shape is the total exterior area of the shape. Shapes such as prisms are made up of flat polygonal surfaces. It is easy to calculate the area of each of these polygons and to then add up the areas to get the surface area of the prism. The units of measurement for surface area are km^2, m^2, cm^2 and mm^2.

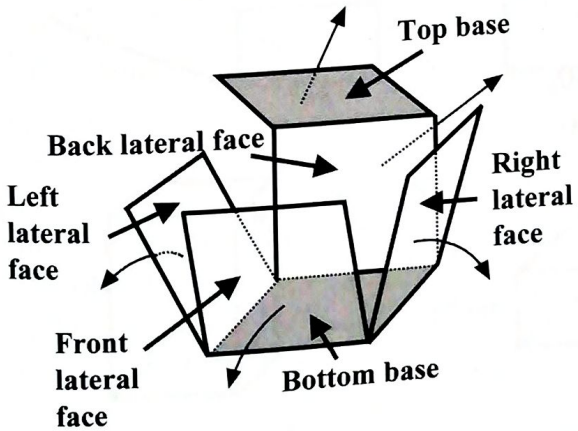
Example 1

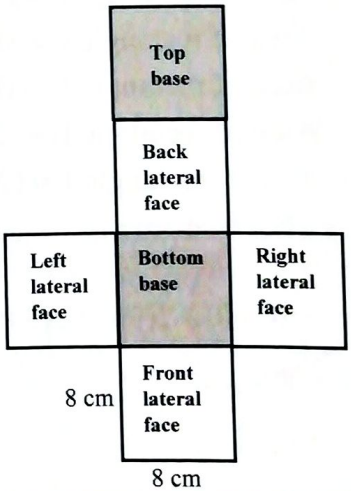
A cube with one side equal to 8 cm is given.
Calculate the surface area of the cube if the cube is closed on all sides.



Solutions

If the cube is opened up and the faces are folded down so that the prism is made into a flat surface, then we call this flat surface a **net**.





The area of one of the squares is $(8)(8) = (8)^2$

There are six congruent squares.

The surface area is the sum of the area of these six squares.

Surface area

$= (8)^2 + (8)^2 + (8)^2 + (8)^2 + (8)^2 + (8)^2$

$= 6(8)^2$

$= 6 \times 64$

$= 384 \text{ cm}^2$

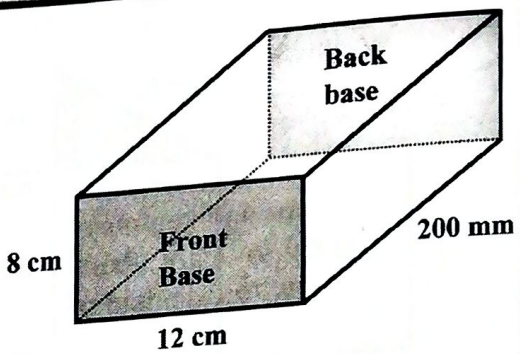
Example 2

A cuboid (rectangular prism) is given.

Calculate the surface area of this cuboid if:

(a) the cuboid is closed on all sides

(b) the cuboid is open at the top and bottom.

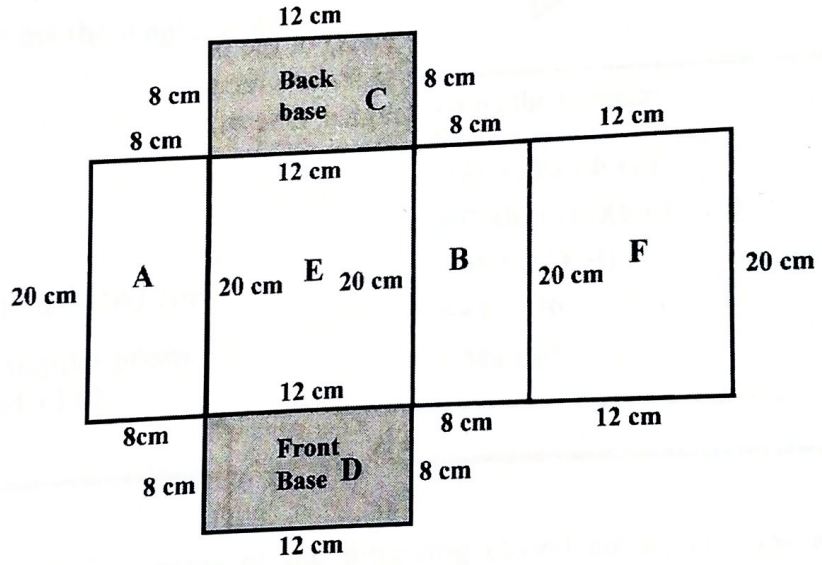


Solutions

(a) First convert 200 mm to cm to ensure that all units are the same.

$200 \text{ mm} \div 10 = 20 \text{ cm}$

Now open up the cuboid and create a net.



The surface area is the sum of the areas of all six rectangles.

Area of rectangle A $= (20)(8) = 160$

Area of rectangle B $= (20)(8) = 160$

Area of rectangle C $= (12)(8) = 96$

Area of rectangle D $= (12)(8) = 96$

Area of rectangle E $= (20)(12) = 240$

Area of rectangle F $= (20)(12) = 240$

$\therefore$ Surface area

$= 2(20)(8) + 2(12)(8) + 2(20)(12)$

$= 2(160) + 2(96) + 2(240)$

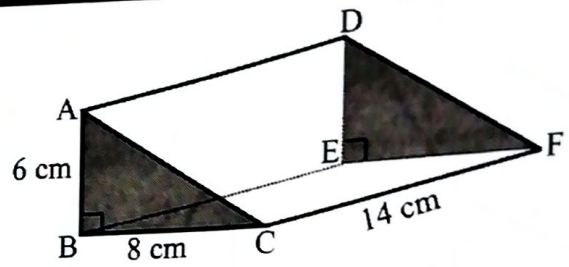
$= 992 \text{ cm}^2$

(b) If the cube is open on the top and bottom, then there are only four rectangles since rectangle E (bottom) and F (top) will be missing. The surface area will then be the sum of the areas of rectangle A, B, C and D.

$\therefore S = 2(160) + 2(96) = 512 \text{ cm}^2$

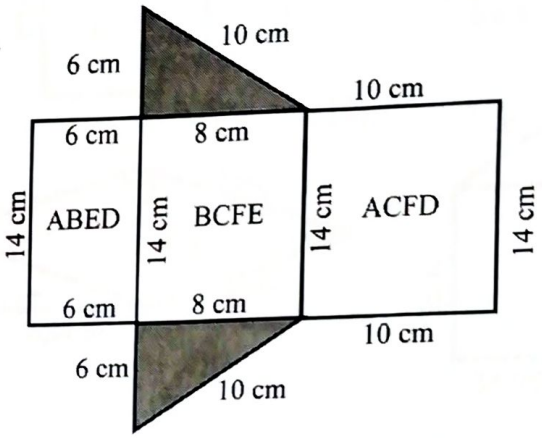
Example 3

A triangular prism is given.
Calculate the surface area of this prism.



Solution

The prism is made up of two triangular bases and three rectangles BCFE, ABED and ACFD.



Area of $\Delta ABC = \frac{1}{2}(8)(6) = 24 \text{ cm}^2$

Area of $\Delta DEF = \frac{1}{2}(8)(6) = 24 \text{ cm}^2$

Area $BCFE = (14)(8) = 112 \text{ cm}^2$

Area $ABED = (14)(6) = 84 \text{ cm}^2$

Use Pythagoras to get the length of AC:

$AC^2 = (6)^2 + (8)^2$

$\therefore AC^2 = 36 + 64$

$\therefore AC^2 = 100$

$\therefore AC = 10 \text{ cm}$

Area $ACFD = (14)(10) = 140 \text{ cm}^2$

Surface area of triangular prism
$= 24 + 24 + 112 + 84 + 140$

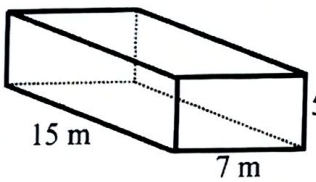
$= 384 \text{ cm}^2$

Using the formula:
Surface area
$= bh + d(a + b + c)$
$= (8)(6) + (14)(6 + 8 + 10)$
$= 48 + (14)(24)$
$= 48 + 336$
$= 384 \text{ cm}^2$

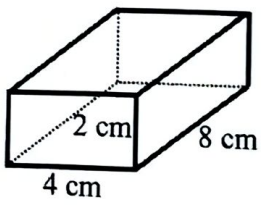
EXERCISE 2

(a) Calculate the surface areas of the following closed cubes, cuboids and triangular prisms.

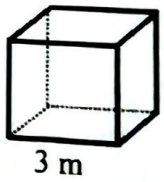
(1)



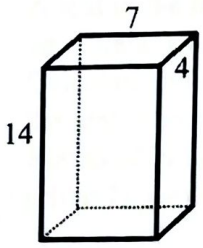
(2)



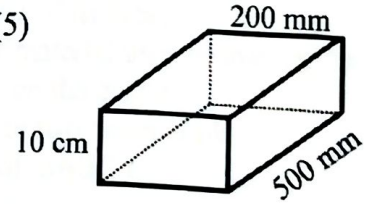
(3)



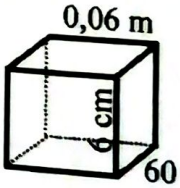
(4)



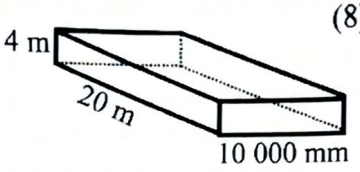
(5)



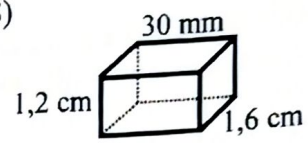
(6)



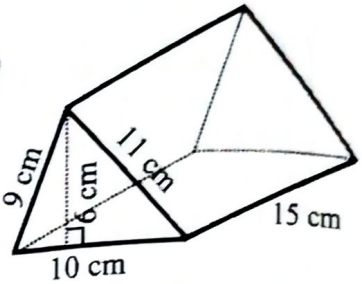
(7)



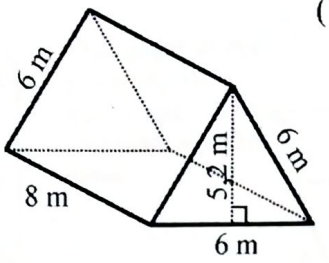
(8)



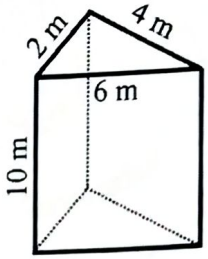
(9)



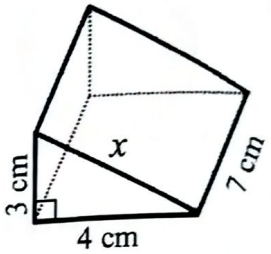
(10)



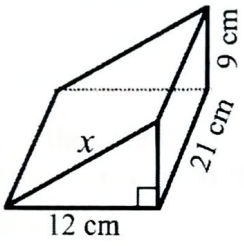
(11)



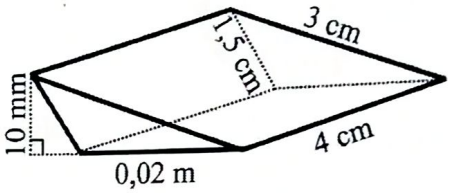
(12)



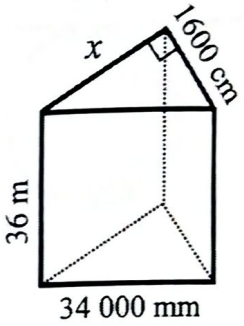
(13)



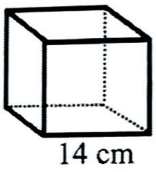
(14)



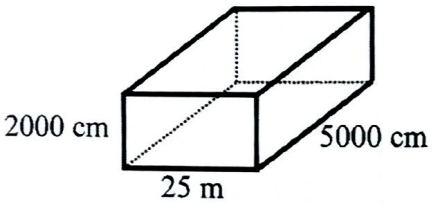
(15)



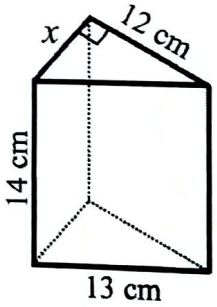
(b) A cube with one side equal to 14 cm is given.
Calculate the surface area of the cube if:
(1) the cube is closed on all sides.
(2) the cube is open on top.



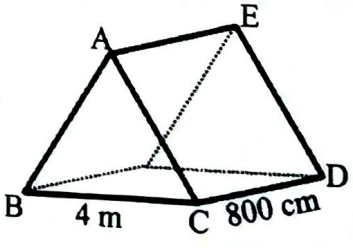
(c) The inside of an olympic-sized pool is to be painted on the inside. Calculate the surface area to be painted.



(d) A triangular prism is open on top. Calculate the surface area of the prism.



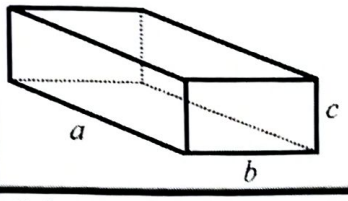
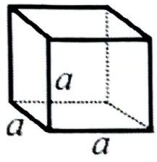
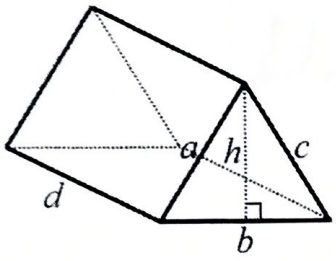
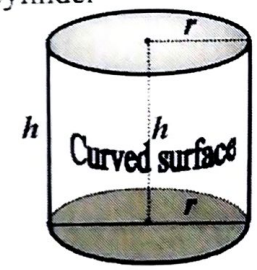
(e) A tent in the form of a triangular prism has an equilateral triangle as one of its faces.
Calculate the amount of material used to manufacture the tent if no material is on the ground.
Round off your answer to two decimal places.
(Hint: Draw the height of ΔABC).



(c) If the surface area of a cuboid is $52\ \text{cm}^2$ and its volume is $24\ \text{cm}^3$, determine the surface area and volume of the cuboid formed if the dimensions of the original cuboid are:

(1) doubled (2) multiplied by 3 (3) multiplied by 5

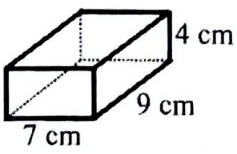
Summary of formulae for calculating surface area and volume

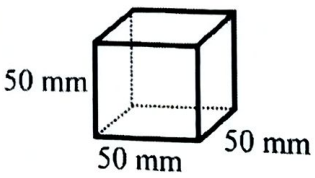
Prism	Surface area	Volume
Cuboid (Rectangular prism) 	Sum of the areas of the six rectangles: Surface area $= 2ab + 2ac + 2bc$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $= (ab) \times c$ $= abc$
Cube 	Sum of the areas of the six squares: Surface area $= 2(a)(a) + 2(a)(a) + 2(a)(a)$ $= 6a^2$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $= a^3$
Triangular prism 	Sum of the areas of two triangles and three rectangles. Surface area $= 2\left[\frac{1}{2}(b \times h)\right] + ad + bd + cd$ $= bh + ad + bd + cd$ $= bh + d(a + b + c)$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $= \frac{1}{2}(b \times h) \times d$ $= \frac{1}{2}bhd$
Cylinder 	Sum of the areas of two circles and a curved surface. Surface area $= 2\pi r^2 + 2\pi rh$	Area of a chosen base multiplied by the distance moved by the base (height). Volume $= \pi r^2 h$

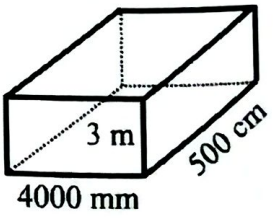
REVISION EXERCISE

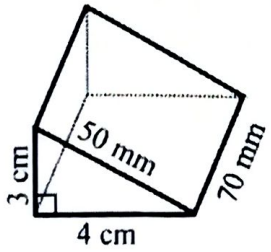
This exercise will also include real-world applications of surface area and volume.

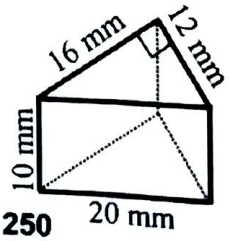
(a) Calculate the surface area and volume of the following prisms.

(1) 

(2) 

(3) 

(4) 

(5) 

(6)